EVALUATION OF THE ACCURACY OF A RELIABILITY MEASUREMENT PROCEDURE USING SIMULATION TECHNIQUES

by

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UNITED STATES
NAVAL POSTGRADUATE SCHOOL

THESIS

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
December 1968

ABSTRACT

This study evaluates the accuracy of an established reliability measurement procedure (NAVWEPS OD 29304) by computer simulation. The reliability measurement procedure assumes components fail according to an Exponential Failure Law. This study tests the accuracy of that procedure when components obey a Weibull Failure Law or a Log Normal Failure Law.

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CHAPTER I

INTRODUCTION

The highly complex weapon systems being developed for the Navy today require a rapid assessment of system and subsystem reliability during research and development, and production phases. Currently there exists a statistical model, the Guide Manual For Reliability Measurement Program (NAVWEPS OD 29304), that "...can be utilized by all contractors for subsystem reliability measurement and by the Navy for weapon system reliability measurement."[1] This model is versatile in that it permits the combination of test data from all levels and can be continuously updated as new test data becomes available. The model is thus a rapid approximation procedure for determining system reliability. However, the procedure has some restrictive assumptions which must be kept in mind.

One notable restrictive assumption is that all components are assumed to have a constant failure rate. This may not be necessarily true, although many times is a good approximation. This leads to the commonly used Exponential Failure Law. However, suppose the failures were actually given by a Weibull or a Log Normal Failure Law. Then this procedure must adequately estimate the system or subsystem

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[1] Guide Manual For Reliability Measurement Program (NAVWEPS OD 29304, 15 May 1965), p. 3-1.

reliability under the Exponential Law assumption.

The purpose of this study is to evaluate the accuracy of the procedure in NAVWEPS OD 29304 when failures actually occur in a Weibull and Log Normal fashion.

The approximation procedure uses statistical estimates of failure rates based on sample data and are thus subject to statistical uncertainty. Therefore, at best, the procedure must yield a lower confidence limit on system or subsystem reliability, or an upper limit on failure rate. It is desired to test the accuracy of this procedure by simulating the distribution of the lower confidence limit for various systems and comparing the simulation results with the true reliability.

The method of simulation involves obtaining random Weibull and Log Normal failure time variates for components in a given system with a true system reliability, R_s. These quantities represent the times to failure of components provided they are less than the duration of the test (i.e. the planned test time). Then, using the statistical procedure in the <u>Guide Manual For Reliability Measurement Program</u>, a $100(1-\alpha)\%$ lower confidence limit, $\hat{R}_{s,L(\alpha)}$, for R_s is obtained. A large number of replications of this process are completed to construct a distribution of $\hat{R}_{s,L(\alpha)}$. If the procedure is indeed accurate, the $(1-\alpha)^{th}$ percentile point of the constructed distribution will be R_s. That is, if $\hat{R}_{s,L(\alpha)}$ is a true $100(1-\alpha)\%$ lower confidence limit for

R_s, then $P(R_s \geq \hat{R}_{s,L(\alpha)}) = 1 - \alpha$. If the $(1 - \alpha)^{th}$ percentile point of the distribution of $\hat{R}_{s,L(\alpha)}$ is denoted by $A_{1-\alpha}$, then $A_{1-\alpha}$ should equal R_s, regardless of the set of parameter values chosen. Thus a measure of the accuracy of the procedure is given by the quantity $|A_{1-\alpha} - R_s|$. Two other measures of the accuracy of this procedure are the mean and standard deviation of the distribution of $\hat{R}_{s,L(\alpha)}$.

The accuracy of this procedure is examined for a variety of combinations of components and sets of parameter values. In all cases the systems under consideration consist of four components in series. All components are assumed to behave independently so the system reliability is the product of the four component reliabilities.

As expected, the closer the Weibull and Log Normal distributions approximate the Exponential, the better the accuracy of the procedure. However, for some components with failure time distributions differing considerably from the Exponential, the accuracy is fairly good using this method of reliability measurement. If the absolute difference $|A_{1-\alpha} - R_s|$ is less than .02, the accuracy is called good. The accuracy of the procedure is good for planned test times of short duration and diminishes as they are increased. The study concludes that the reliability measurement procedure, being an approximation procedure, is useful for some systems following Weibull and Log Normal Failure Laws provided they do not deviate from the Exponential Failure Law by too significant an amount.

CHAPTER II

STATISTICAL RELIABILITY MODEL[2]

The statistical Reliability Model provides the guide lines for reliability measurement on Navy weapons systems. The "Guide Manual" can be used by both contractors for subsystem reliability measurement and by the Navy for weapon system reliability measurement. A brief description of this model's assumptions and methodology are presented in this chapter.

The system reliability obtained is based upon testing components for a time duration, called the planned test time, or until failure occurs under a given stress condition. Also, the model provides for testing a system for component failure by the same method.

This model considers components in a complex system which must operate successfully over a defined "mission", lasting a specified duration of time. Time can thus be measured in "mission units". System reliability is thus a function of environmental and usage stresses which include vibration, shock, etc., and the operating vs. non-operating conditions as well as a function of the associated part failure rate parameters together with the time duration of the environments and usage stresses. The reliability

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[2]<u>Guide Manual For Reliability Measurement Program</u> (NAVWEPS OD 29304, 15 May 1965), pp. 3-1 to 3-15.

measurement system is dependent upon the following assumptions:

1. Constant failure rate. The exponential failure law is assumed to hold, mainly for its common usage, mathematical simplicity, practical simplicity, and reasonability.

2. Additivity of stress effects. "The failure rate induced by two simultaneously acting stresses is equal to the sum of the failure rates due to the two stresses acting sequentially."[3] Thus, mission experience can be simulated by adding data from separate environmental tests.

3. Independence of component failures. "...This is assumed because components are normally tested individually by type, and subsystem reliability is estimated using component and other applicable test results."[4]

4. Failure rate constancy. "The failure rate is considered a function of only the stress acting."[5] In other words, the component being tested has no "memory" as to previous stresses of a different type.

Computation of the system reliability is accomplished by the following procedure:

An unbiased failure rate estimate is computed, providing the failure rates are anticipated as being small. For ease of computation, a simplified unbiased estimator, which is slightly conservative, is used. For the i^{th} component, the unbiased estimator, $\hat{\lambda}_i$, is given by:

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[3] Ibid. p.3-3.

[4] Ibid.

[5] Ibid.

$$\hat{\lambda}_i = \frac{f_i}{\sum_{j=1}^{N_i} T_{ij}} \cdot \frac{2N_i}{2N_i + 1} \tag{1}$$

where N_i = the sample size of the i^{th} component (i.e. the number of components of type i tested).

T_i = the test time (time to failure) of the component of type i.

T_{oi} = the planned test time for component i.

and f_i = the number of N_i failure times which are less than T_{oi}.

The system failure rate estimate is then given by:

$$\hat{\lambda} = \sum_{i=1}^{m} \hat{\lambda}_i \tag{2}$$

where m is the number of components in the system. The component number assumes values i = 1,...,m.

The variance of the unbiased failure rate estimate is given by:

$$\text{Variance} = \sum_{i=1}^{m} (\hat{\lambda}_i / S_i)$$

where

$$S_i = \sum_{j=1}^{N_i} T_{ij} = \text{the sum of all test times}$$

accumulated on the N components of type i.

These estimates of failure rates for components in a given test condition can be continually updated during the development phase as more data becomes available. Also, during any higher level of assemblage, further testing can also yield data to modify the failure rate estimate. Data is obtained indicating the time that each component operates during the system test.

The reliability estimate for the system is then given by $\hat{R}_s = e^{-\hat{\lambda}}$ where $\hat{\lambda}$ is the appropriate failure rate estimate and only series systems or subsystems are considered, given by formula (2).

The reliability estimates obtained are subject to statistical uncertainty, therefore the important item to observe is the confidence interval about the estimate. Here, the lower confidence limit on system reliability becomes the pertinent quantity to observe. The "Guide Manual" bases the lower confidence limit on Normal theory and it is corrected to compensate for small values of λ. The upper limit on failure rate, $\hat{\lambda}_u$, provides the corresponding lower confidence limit on reliability by $\hat{R}_{s,L} = e^{-\hat{\lambda}_u}$. The upper limit on failure rate is given by:

$$\hat{\lambda}_u = \begin{cases} \dfrac{2\hat{\lambda} + K^2\hat{c} + \sqrt{4\hat{\lambda}K^2\hat{c} + K^4\hat{c}^2}}{2} & \text{for } \sum_{i=1}^{m} f_i > 0 \\ \dfrac{K^2}{n} \sum_{i=1}^{m} (1/S_i) & \text{for } \sum_{i=1}^{m} f_i = 0 \end{cases} \qquad (3)$$

where

$$\hat{c} = \frac{\sum_{i=1}^{m} (\hat{\lambda}_i / S_i)}{\hat{\lambda}}$$

n = the number of component-environment-test condition terms in the summation, and K = the percentage point of the Normal Distribution. The values of K for a given confidence level are not completely appropriate for small values of λ. Consequently, a correction factor, β, is used to obtain desired precision. Beta values are tabulated for the 80% confidence limit in reference (2). Thus the percentage point of the Normal Distribution, K, is modified by βK and replaced in formula (3).

A more detailed description of this reliability measurement method plus examples of the procedure are given in reference (2).

CHAPTER III

SIMULATION PROCEDURE

Suppose the assumption of constant failure rate used in the previously described statistical model is relaxed. It is now desired to test the accuracy of that model when failures occur according to a Weibull Failure Law and a Log Normal Failure Law. For testing purposes by simulation techniques, consider a series system with four components, each component having a well defined failure law.

The true reliability for each component of this series system is given by R_i, $i = 1,2,3,4$, and thus the true system reliability is

$$R_s = \sum_{i=1}^{4} R_i \quad . \tag{4}$$

Using the statistical model, an estimate of the $(1-\alpha)^{th}$ percentile confidence limit of the system, $\hat{R}_{s,L(\alpha)}$, can be obtained by computer simulation. This is a random variable as calculated from the statistical model. If

$$P(\ R_s \geq \hat{R}_{s,L(\alpha)}\ ) = 1-\alpha$$

then, in fact, $\hat{R}_{s,L(\alpha)}$ is an exact $(1-\alpha)^{th}$ percentile lower confidence limit for R_s. This says that R_s is always the $(1-\alpha)^{th}$ percentile point of the probability distribution of $\hat{R}_{s,L(\alpha)}$. The distribution of $\hat{R}_{s,L(\alpha)}$ is constructed by

computer simulation. Letting $A_{1-\alpha}$ be the $(1-\alpha)^{th}$ percentile point of the distribution of $\hat{R}_{s,L(\alpha)}$, then the absolute difference, $|A_{1-\alpha} - R_s|$, is a measure of the accuracy of the statistical model.

Two other measures of the accuracy of this model are the estimated mean, $\overline{\hat{R}}_{s,L(\alpha)}$, and the estimated standard deviation, $s_{\hat{R}_{s,L(\alpha)}}$, which give some assurance that the actual values generated by the procedure are reasonable.

The simulation procedure itself is now discussed for the series system of four components:

The distribution of $\hat{R}_{s,L(\alpha)}$ is constructed by generating 500 random observations on $\hat{R}_{s,L(\alpha)}$ for a given system, number of components tested, and reliability of the components. For this study a four component series system is used, with each component having reliability R_i, $i = 1,2,3,4$. The reliability, R_i, is defined for each component by $R_i = R_i(1) = P(\ T_i > 1)$, where T_i is the time to failure random variable for a component of type i. This establishes the parameter values for a particular failure rate distribution. For example, if a component is to obey a Weibull Failure Law,

$$R(t) = e^{-(\lambda t)^{\beta}} \tag{5}$$

and $R(1) = e^{-\lambda^{\beta}} = .995$, then, for $\beta = 2$, this implies that $\lambda = .0707$.

Similarly, for the Log Normal Failure Law, if

$$R(t) = e^{Z} = 1 - \Phi\left[\frac{\ln t - \mu}{\sigma}\right] \tag{6}$$

and $R(1) = 1 - \Phi(-\mu/\sigma) = .995$, then, for $\sigma = 1$, this implies that $\mu = 2.576$. The failure rate functions determined from the reliabilities and selected parameters are displayed in Appendix I.

A random number generator is used to obtain a sample of N_i tested components of type i which fail according to their respective failure rate distributions. This Monte Carlo method draws a random number which is Uniformly Distributed (0,1) and converts it to a Weibull variate or a Log Normal variate to obtain the time to failure of the i^{th} component, T_i. For the Weibull Distribution, $e^{-(\lambda T)^{\beta}} = Y$, where Y is distributed as Uniform (0,1), implies that

$$T = \frac{(-\ln Y)^{1/\beta}}{\lambda} \tag{7}$$

is a random Weibull variate. For the Log Normal Distribution, if X is a random variable distributed as Normal(0,1), then $Z = \sigma X + \mu$ is a random variable distributed as Normal (μ, σ^2) and thus $T = e^{Z}$ is a random Log Normal variate. In this manner, N_i failure times are generated for each of the four components:

$$\begin{array}{lllll}
T_{11} & T_{12} & T_{13} & \cdots & T_{1N_1} \\
T_{21} & T_{22} & T_{23} & \cdots & T_{2N_2} \\
T_{31} & T_{32} & T_{33} & \cdots & T_{3N_3} \\
T_{41} & T_{42} & T_{43} & \cdots & T_{4N_4}
\end{array}$$

Using these failure times in the statistical procedure discussed in the previous chapter, values of $\hat{R}_s$ and $\hat{R}_{s,L(\alpha)}$ are obtained with that model. Each T_{ij} is examined for failure before the end of the planned test time or else terminated at the end of the planned test time. This same procedure is completed 500 times to generate 500 random observations, thus constructing the distributions of $\hat{R}_s$ and $\hat{R}_{s,L(\alpha)}$.

The mean and standard deviation of the distributions are determined by:

$$\bar{\hat{R}}_{s,L(\alpha)} = \frac{1}{500} \sum_{k=1}^{500} \hat{R}_{s,L(\alpha)_k} \qquad (8)$$

$$\bar{\hat{R}}_s = \frac{1}{500} \sum_{k=1}^{500} \hat{R}_{s_k} \qquad (9)$$

$$s_{\hat{R}_{s,L(\alpha)}} = \sqrt{\frac{1}{500} \sum_{k=1}^{500} \left(\hat{R}_{s,L(\alpha)_k} - \bar{\hat{R}}_{s,L(\alpha)}\right)^2} \qquad (10)$$

$$s_{\hat{R}_s} = \sqrt{\frac{1}{500} \sum_{k=1}^{500} \left(\hat{R}_{s_k} - \bar{\hat{R}}_s\right)^2} \qquad (11)$$

The 500 values of $\hat{R}_{s,L(\alpha)}$ are then ordered and the $(1-\alpha)^{th}$ percentile point of the distribution is obtained. Thus, the value $A_{1-\alpha}$ is available for comparison with R_s. This process is done merely by counting down to the 101^{st} value of $\hat{R}_{s,L(\alpha)}$ from the largest, which represents the 80^{th} percentile point.

Several cases are examined with various failure rate distributions and with various combinations of components. For each case examined, the planned test time, T_{oi}, varies to produce an optimistic or pessimistic reliability test. If $T_{oi} = .5$, the system will yield an optimistic test since the planned test time will be reached before most components fail. The opposite will occur when $T_{oi} = 5$. A T_{oi} chosen such that the average failure rate is equal to the failure rate at time one appears to be a good test of system reliability for this procedure. That is, make T_{oi} such that it satisfies the equation

$$\frac{1}{T_{oi}} \int_{0}^{T_{oi}} z_i(t)dt = z_i(1) \tag{12}$$

where z(t) is the failure rate function; the failure distribution divided by the reliability function: $z(t) = f(t)/R(t)$.

All cases studied by simulation procedures are given in Table I. All simulation results of these cases are given in Table II.

TABLE I

CASES STUDIED BY SIMULATION PROCEDURES

Case No.	Component No.	Failure Law	Parameters	R_i	R_s
1	1	Weibull	β=1.5, λ=.0293	.995	.98
	2	Weibull	β=2.0, λ=.0707	.995	
	3	Log Normal	σ=1.0, μ=2.576	.995	
	4	Log Normal	σ=2.0, μ=5.152	.995	
2	1	Weibull	β=1.5, λ=.0293	.995	.98
	2	Weibull	β=1.5, λ=.0293	.995	
	3	Log Normal	σ=2.0, μ=5.152	.995	
	4	Log Normal	σ=2.0, μ=5.152	.995	
3	1	Weibull	β=1.33, λ=.0378	.987	.95
	2	Weibull	β=1.33, λ=.0378	.987	
	3	Log Normal	σ=1.5, μ=3.345	.987	
	4	Log Normal	σ=1.5, μ=3.345	.987	
4	1	Weibull	β=1.2, λ=.0485	.974	.90
	2	Weibull	β=1.2, λ=.0485	.974	
	3	Log Normal	σ=1.5, μ=2.919	.974	
	4	Log Normal	σ=1.5, μ=2.919	.974	
5	1	Weibull	β=1.2, λ=.0485	.974	.90
	2	Weibull	β=1.2, λ=.0485	.974	
	3	Weibull	β=1.2, λ=.0485	.974	
	4	Weibull	β=1.2, λ=.0485	.974	

TABLE I (Continued)

Case No.	Component No.	Failure Law	Parameters	R_i	R_s
6	1	Log Normal	$\sigma=1.5, \mu=2.919$	.974	.90
	2	Log Normal	$\sigma=1.5, \mu=2.919$	.974	
	3	Log Normal	$\sigma=1.5, \mu=2.919$	.974	
	4	Log Normal	$\sigma=1.5, \mu=2.919$	.974	
7	1	Weibull	$\beta=1.1, \lambda=.0483$	.965	.867
	2	Weibull	$\beta=1.1, \lambda=.0483$	.965	
	3	Weibull	$\beta=1.1, \lambda=.0483$	.965	
	4	Weibull	$\beta=1.1, \lambda=.0483$	.965	
8	1	Log Normal	$\sigma=2.0, \mu=4.0$	.977	.912
	2	Log Normal	$\sigma=2.0, \mu=4.0$	.977	
	3	Log Normal	$\sigma=2.0, \mu=4.0$	.977	
	4	Log Normal	$\sigma=2.0, \mu=4.0$	.977	

CHAPTER IV

RESULTS AND CONCLUSIONS

In the preceding chapters the statistical procedure from the <u>Guide Manual For Reliability Measurement Program</u> has been briefly explained to obtain a $100(1-\alpha)\%$ lower confidence limit on system reliability. The simulation of this method on a computer has been explained and the technique for measuring the accuracy of the statistical procedure stated. Now, from the results of the simulation, some conclusions may be drawn.

The accuracy of the statistical procedure has been examined for eight cases which represent different combinations of components for various sets of parameter values. The results for these cases are given in Table II. The mean and standard deviation of the 500 values of $\hat{R}_{s,L(\alpha)}$ and $\hat{R}_s$ are given, along with the $(1-\alpha)^{th}$ percentile point, $A_{1-\alpha}$. In all cases studied, α was taken to be .20.

The accuracy of the procedure is measured by the absolute difference between the 80^{th} percentile of the distribution of $\hat{R}_{s,L(.20)}$, which is $A_{.80}$, and the actual system reliability, R_s (i.e. accuracy $= \left|A_{.80} - R_s\right|$).

Also stated in the results, is the quantity TT, which is the amount of testing relative to the component unreliabilities. In other words, this is a measure of the amount of testing required to achieve a desired accuracy for a given system with component failure rates, $z_i(1)$,

and planned test times, T_{oi}. Then TT is given by

$$TT = \sum_{i=1}^{m} N_i z_i(1) T_{oi} \ . \tag{13}$$

As an example of the accuracy study, consider Case 1, where the system consists of four components in series displaying the following failure laws:

Component 1: Weibull, $\beta = 1.5$, $\lambda = .0293$
Component 2: Weibull, $\beta = 2.0$, $\lambda = .0707$
Component 3: Log Normal, $\sigma = 1.0$, $\mu = 2.576$
Component 4: Log Normal, $\sigma = 2.0$, $\mu = 5.152$

The component reliabilities, $R_i = .995$, for all four components, so the system reliability, $R_s = .9801$. The measure of accuracy for the optimistic planned test time, $T_{oi} = .5$, is $\left| A_{.80} - R_s \right| = .008$ for 500 components of each type tested. The pessimistic planned test time, $T_{oi} = 5.0$, yields an accuracy of .055 for 100 components of each type tested. The <u>ad hoc</u> planned test times, $T_{o1} = 2.25$, $T_{o2} = 2.0$, $T_{o3} = 1.95$, and $T_{o4} = 3.25$, as determined from formula (12), give an accuracy of .030 for $N_i = 100$. The accuracy for the <u>ad hoc</u> planned test times is only fair. This planned test time represents the best measure of accuracy of the three selected.

It can be seen from Table II and Appendix I that the accuracy of the procedure is a function of how closely the Weibull and Log Normal failure rate functions approximate a constant failure rate function. The greater the Weibull

and Log Normal failure rate functions deviate from the constant failure rate, the more inaccurate the procedure. Also, the accuracy decreases as the planned test time is increased. The Weibull failure rate function increases in time and the Log Normal failure rate function increases then decreases in time, thus deviating from the constant failure rate for large planned test times.

It can thus be concluded from the study that the statistical procedure can be useful for reliability approximations of a system, if the components that fail according to these non-constant rates do not deviate too significantly from a constant failure rate.

TABLE II

SIMULATION RESULTS

CASE 1 $R_s = .9801$

		$\hat{R}_{s,L(\alpha)}$		$\hat{R}_s$				
T_{oi}	N_i	mean	std.dev.	mean	std.dev.	$A_{.80}$	accuracy	TT
.5	50	.9258	.0244	.9908	.0193	.9376	.043	.5
	100	.9565	.0175	.9910	.0135	.9683	.012	1.0
	500	.9817	.0074	.9908	.0060	.9881	.008	5.0
5.0	50	.9070	.0178	.9246	.0165	.9202	.060	5.0
	100	.9136	.0129	.9243	.0123	.9256	.055	10.
	500	.9190	.0052	.9236	.0051	.9287	.052	50.
2.25, i = 1 2.00, i = 2 1.95, i = 3 3.25, i = 4	50	.9383	.0218	.9621	.0185	.9577	.033	2.5
	100	.9469	.0155	.9618	.0137	.9598	.030	5.0
	500	.9567	.0059	.9619	.0055	.9620	.028	25.
1.63, i = 1 1.50, i = 2 1.47, i = 3 2.13, i = 4	100	.9545	.0159	.9712	.0136	.9679	.012	2.4
	200	.9604	.0105	.9710	.0093	.9721	.008	4.7
	300	.9629	.0084	.9711	.0076	.9702	.010	7.1

TABLE II (Continued)

CASE 2 $R_s = .9801$

		$\hat{R}_{s,L(\alpha)}$		$\hat{R}_s$				
T_{oi}	N_i	mean	std.dev.	mean	std.dev.	$A_{.80}$	accuracy	TT
.5	50	.9248	.0255	.9905	.0225	.9355	.045	.5
	100	.9451	.0159	.9902	.0146	.9621	.018	1.0
	500	.9783	.0083	.9880	.0068	.9881	.008	5.0
5.0	50	.9499	.0137	.9636	.0122	.9636	.017	5.0
	100	.9542	.0094	.9631	.0087	.9618	.018	10.
	500	.9596	.0038	.9629	.0036	.9686	.012	50.
2.25, i = 1 2.25, i = 2 3.25, i = 3 3.25, i = 4	50	.9524	.0178	.9715	.0148	.9678	.012	2.8
	100	.9593	.0125	.9713	.0109	.9701	.010	5.5
	500	.9670	.0047	.9711	.0044	.9710	.009	27.5

TABLE II (Continued)

CASE 3 $R_s = .95$

		$\hat{R}_{s,L(\alpha)}$		$\hat{R}_s$				
T_{oi}	N_i	mean	std.dev.	mean	std.dev.	$A_{.80}$	accuracy	TT
.5	50	.9000	.0417	.9701	.0340	.9376	.012	1.3
	100	.9287	.0304	.9689	.0249	.9683	.018	2.5
	500	.9552	.0123	.9681	.0109	.9643	.014	12.7
5.0	50	.8903	.0188	.9089	.0177	.9070	.043	12.7
	100	.8969	.0138	.9084	.0131	.9090	.041	25.4
	500	.9034	.0057	.9084	.0055	.9101	.040	127.
2.37, i = 1 2.37, i = 2 2.72, i = 3 2.72, i = 4	50	.9002	.0254	.9262	.0229	.9220	.028	2.6
	100	.9074	.0180	.9245	.0168	.9235	.027	5.2
	500	.9178	.0073	.9242	.0070	.9312	.019	25.8

TABLE II (Continued)

CASE 4 $R_s = .90$

		$\hat{R}_{s,L(\alpha)}$		$\hat{R}_s$				
T_{oi}	N_i	mean	std.dev.	mean	std.dev.	$A_{.80}$	accuracy	TT
.5	50	.8528	.0620	.9303	.0536	.8876	.012	2.6
	100	.8805	.0432	.9279	.0379	.9179	.018	5.2
	500	.9110	.0171	.9279	.0160	.9257	.026	26.0
5.0	50	.8339	.0232	.8542	.0224	.8537	.046	26.0
	100	.8405	.0163	.8544	.0157	.8545	.045	52.0
	500	.8479	.0070	.8540	.0069	.8542	.046	260.
2.49, i = 1 2.49, i = 2 3.70, i = 3 3.70, i = 4	50	.8375	.0287	.8655	.0267	.8617	.038	16.1
	100	.8468	.0204	.8640	.0194	.8640	.036	32.2
	500	.8563	.0084	.8638	.0082	.8693	.031	161.

TABLE II (Continued)

CASE 5 $R_s = .90$

		$\hat{R}_{s,L(\alpha)}$		$\hat{R}_s$				
T_{oi}	N_i	mean	std.dev.	mean	std.dev.	$A_{.80}$	accuracy	TT
1	50	.8494	.0452	.9007	.0406	.8954	.005	5.2
	100	.8670	.0307	.8999	.0282	.9347	.035	10.4
2.49	50	.8517	.0302	.8823	.0280	.8772	.023	12.9
	100	.8627	.0197	.8818	.0194	.8789	.021	25.9
	500	.8730	.0090	.8808	.0090	.8865	.014	129.
5	50	.8459	.0221	.8658	.0215	.8633	.037	26.0
	100	.8522	.0154	.8657	.0148	.8655	.035	52.0
10	50	.8338	.0170	.8486	.0163	.8478	.052	52.0
	100	.8383	.0121	.8486	.0118	.8485	.052	104.
15	50	.8259	.0146	.8389	.0140	.8379	.062	78.0
	100	.8298	.0103	.8389	.0100	.8387	.061	156.
20	50	.8204	.0134	.8325	.0129	.8324	.068	104.
	100	.8237	.0097	.8321	.0092	.8314	.069	208.

TABLE II (Continued)

CASE 6 $R_s = .90$

		$\hat{R}_{s,L(\alpha)}$		$\hat{R}_s$				
T_{oi}	N_i	mean	std.dev.	mean	std.dev.	$A_{.80}$	accuracy	TT
.5	50	.8712	.0549	.9460	.0465	.9376	.038	2.6
	100	.8992	.0370	.9442	.0317	.9416	.042	5.2
	500	.9271	.0153	.9428	.0141	.9383	.038	26.0
1.0	50	.8546	.0468	.9051	.0419	.8952	.005	5.2
	100	.8712	.0322	.9036	.0295	.9131	.013	10.4
	500	.8914	.0135	.9029	.0129	.9041	.004	52.0
3.70	50	.8227	.0264	.8478	.0260	.8458	.054	19.2
	100	.8307	.0188	.8471	.0181	.8464	.054	38.4
	500	.8400	.0089	.8471	.0087	.8470	.053	192.

TABLE II (Continued)

CASE 7 $R_s = .867$

		$\hat{R}_{s,L(\alpha)}$		$\hat{R}_s$				
T_{oi}	N_i	mean	std.dev.	mean	std.dev.	$A_{.80}$	accuracy	TT
1	50	.8132	.0502	.8678	.0462	.8736	.006	7.0
	100	.8321	.0340	.8675	.0319	.9234	.056	14.0
5	50	.8263	.0237	.8469	.0228	.8461	.021	35.0
	100	.8325	.0165	.8467	.0159	.8464	.021	70.0
10	50	.8222	.0178	.8376	.0171	.8370	.030	70.0
	100	.8270	.0126	.8377	.0122	.8371	.030	140.
15	50	.8195	.0151	.8328	.0146	.8323	.035	105.
	100	.8233	.0107	.8326	.0104	.8323	.035	210.
20	50	.8170	.0140	.8293	.0134	.8295	.038	140.
	100	.8205	.0098	.8290	.0095	.8283	.039	280.

TABLE II (Continued)

CASE 8 $R_s = .912$

		$\hat{R}_{s,L(\alpha)}$		$\hat{R}_s$				
T_{oi}	N_i	mean	std.dev.	mean	std.dev.	$A_{.80}$	accuracy	TT
1	50	.8674	.0451	.9166	.0399	.8959	.016	4.6
	100	.8839	.0313	.9152	.0284	.9348	.023	9.2
2	50	.8707	.0329	.9035	.0300	.9006	.011	9.2
	100	.8813	.0223	.9028	.0210	.9075	.004	18.4
5	50	.8853	.0195	.9044	.0185	.9013	.011	23.0
	100	.8926	.0130	.9043	.0124	.9034	.009	46.0
10	50	.9021	.0143	.9138	.0135	.9158	.004	46.0
	100	.9054	.0099	.9135	.0095	.9145	.003	92.0
15	50	.9118	.0118	.9211	.0111	.9228	.011	69.0
	100	.9143	.0083	.9208	.0079	.9216	.010	138.
20	50	.9187	.0101	.9267	.0096	.9279	.016	92.0
	100	.9209	.0072	.9264	.0069	.9273	.015	184.
100	50	.9532	.0050	.9566	.0047	.9577	.046	460.
	100	.9543	.0034	.9567	.0033	.9571	.045	920.

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2. Guide Manual For Reliability Measurement Program (NAVWEPS OD 29304), 15 May 1965.

3. Meyer, Paul L., Introductory Probability and Statistical Applications, Addison-Wesley, Reading Mass., 1965.

4. Parzen, Emanuel, Modern Probability Theory and Its Applications, Wiley, New York, 1965.

APPENDIX I

GRAPHS OF COMPONENT FAILURE RATE FUNCTIONS FOR EACH SYSTEM TESTED

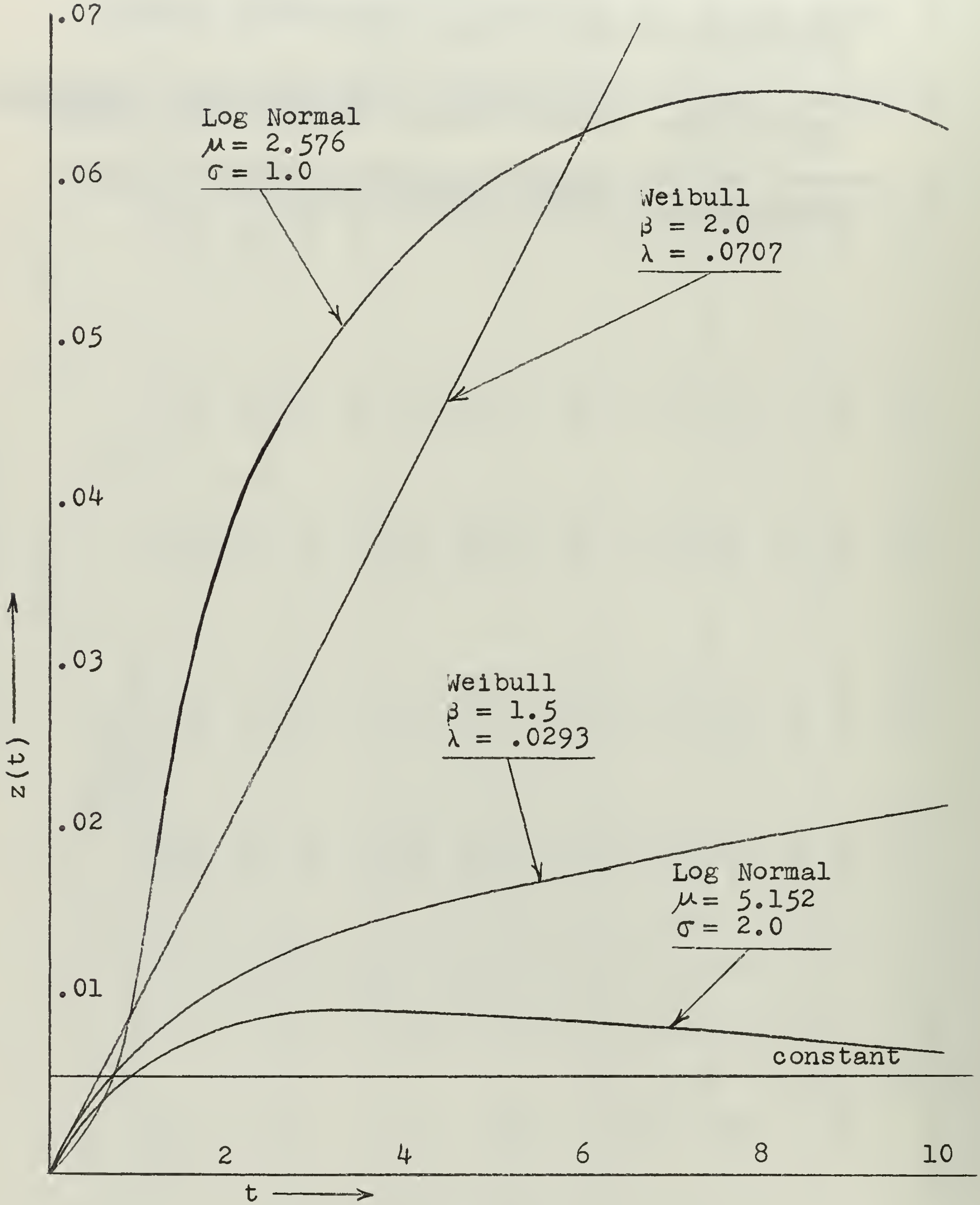

FIGURE 1

Component Failure Rate Functions, z(t), for Case 1.

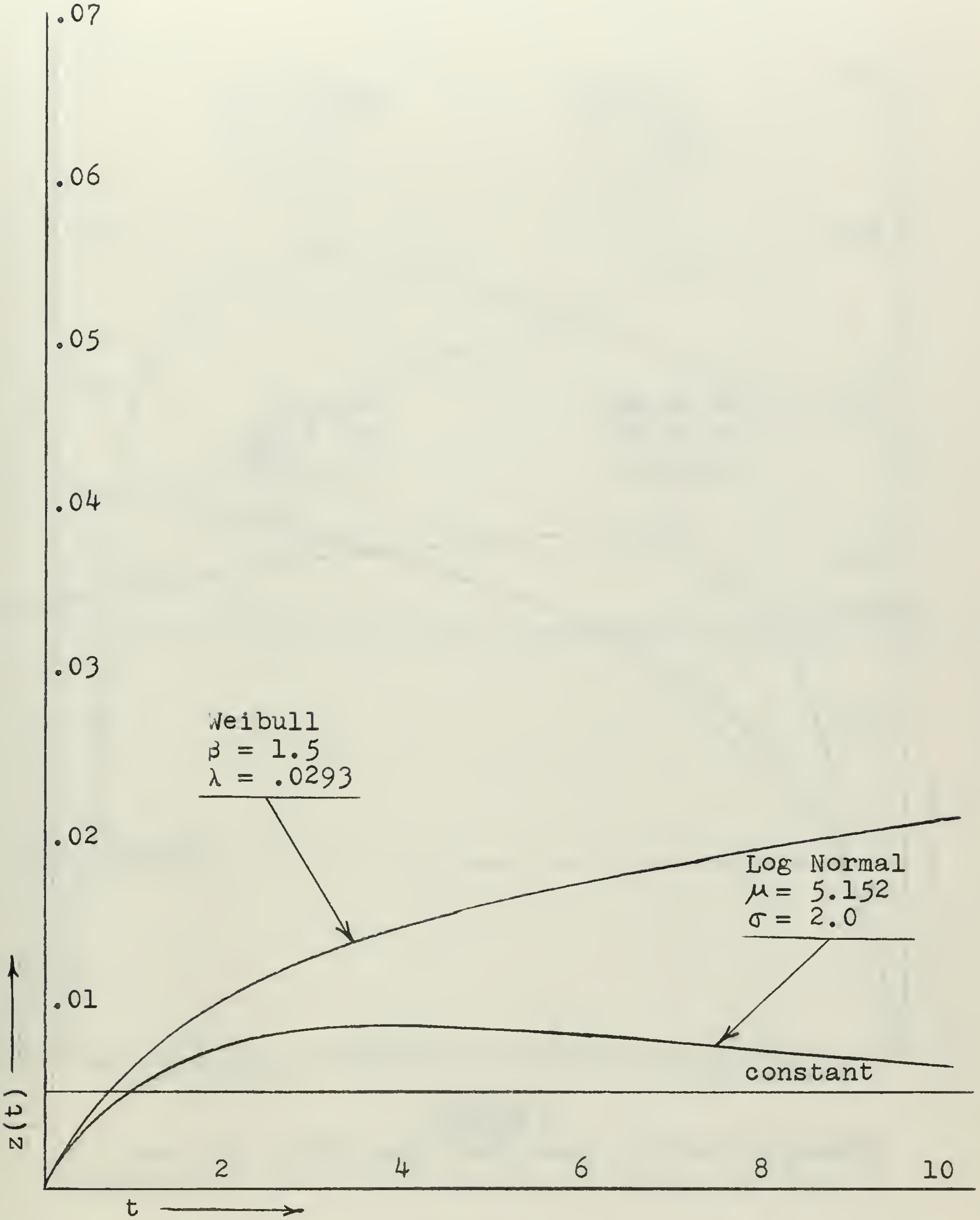

FIGURE 2

Component Failure Rate Functions, z(t), for Case 2.

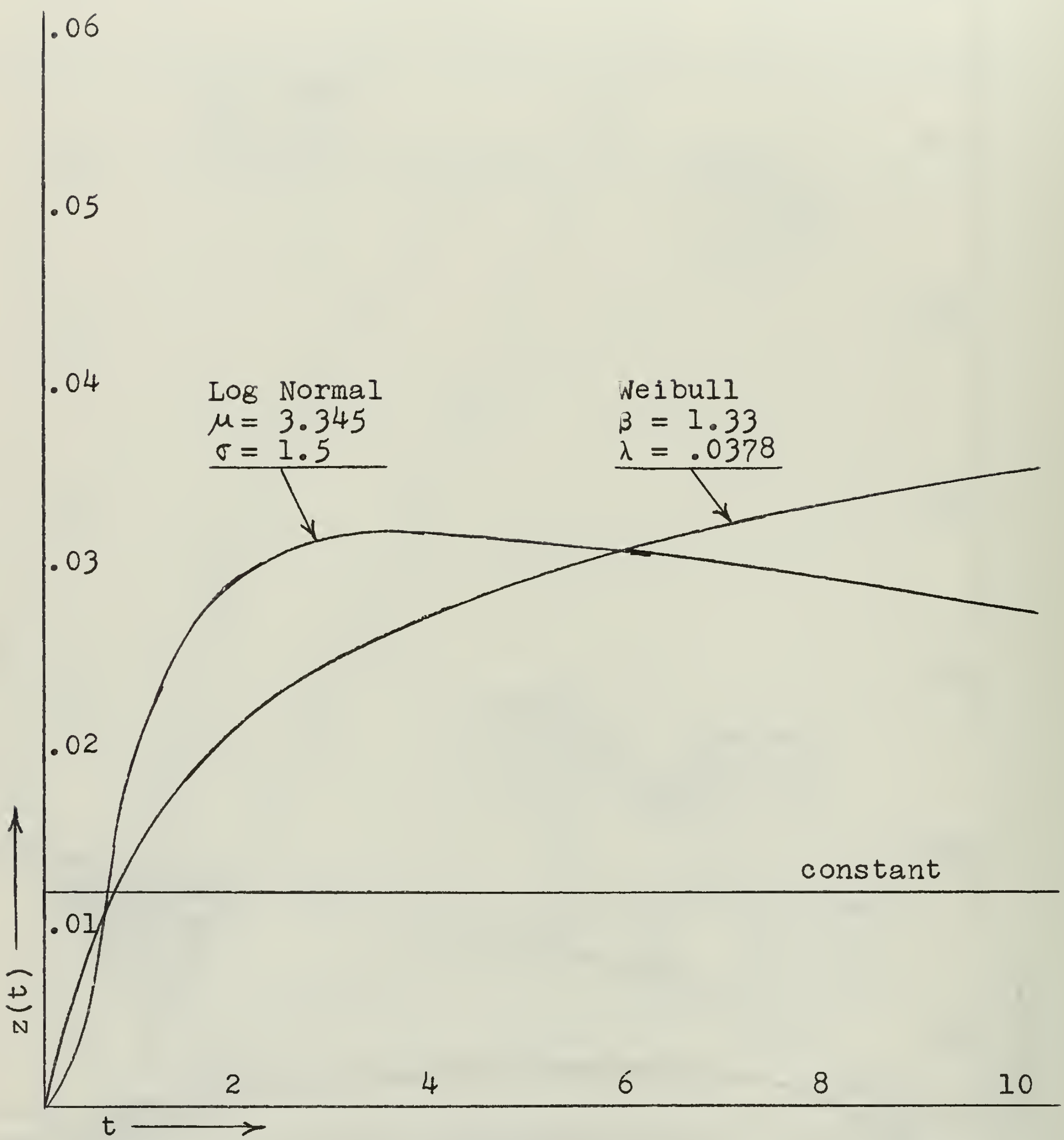

FIGURE 3

Component Failure Rate Functions, z(t), for Case 3.

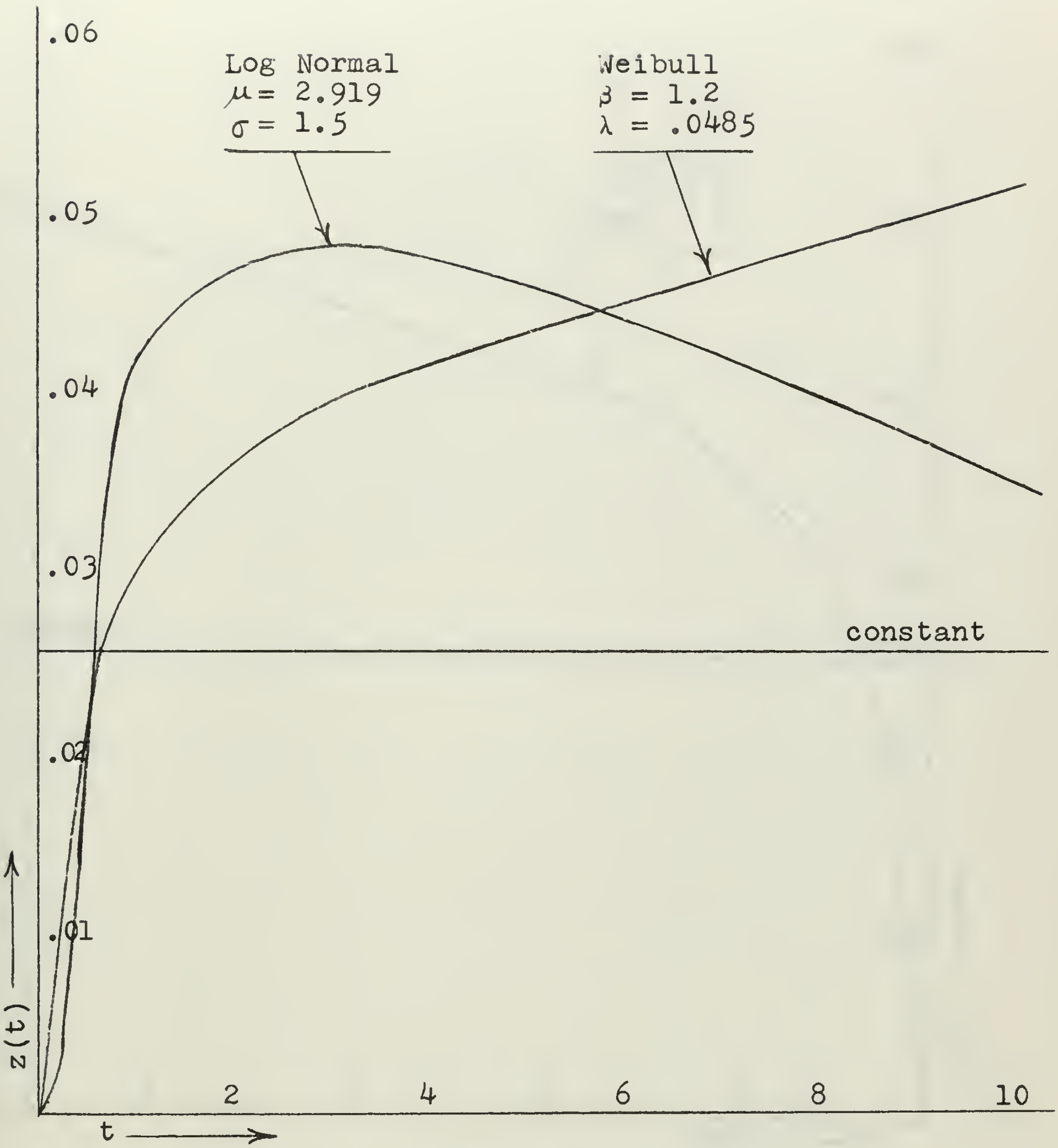

FIGURE 4

Component Failure Rate Functions, z(t), for Case 4.

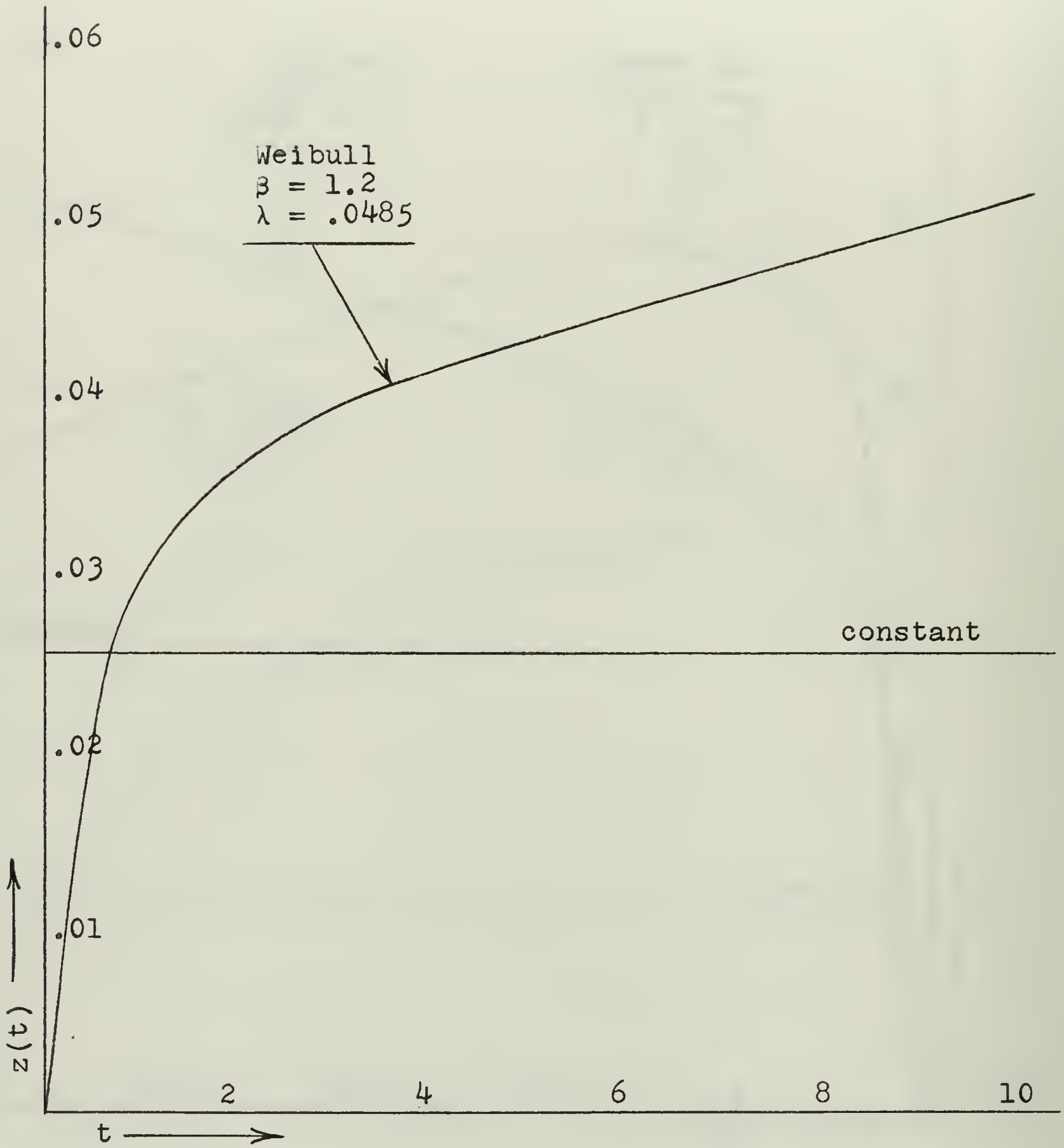

FIGURE 5

Component Failure Rate Functions, z(t), for Case 5.

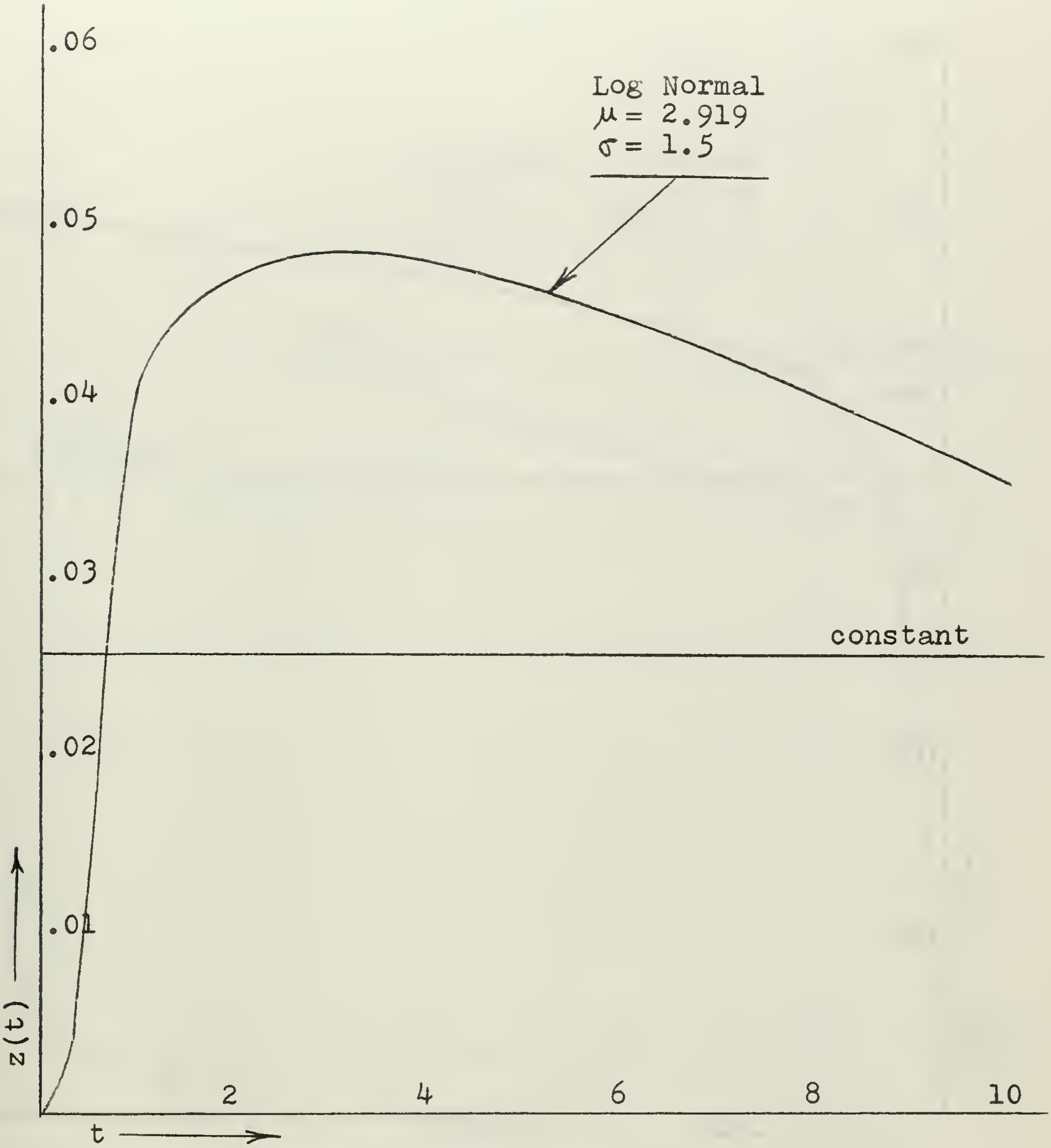

FIGURE 6

Component Failure Rate Functions, z(t), for Case 6.

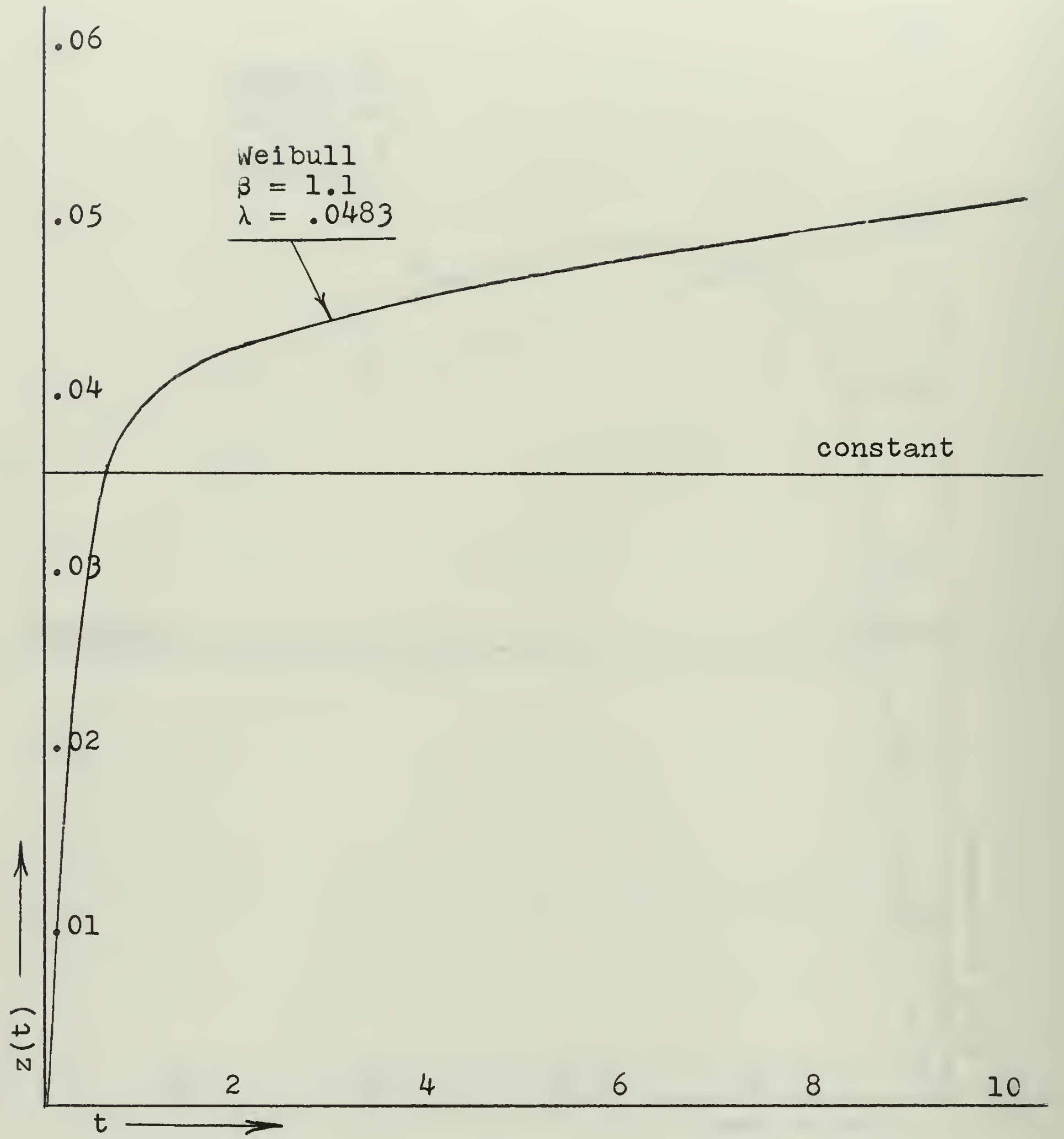

FIGURE 7

Component Failure Rate Functions, z(t), for Case 7.

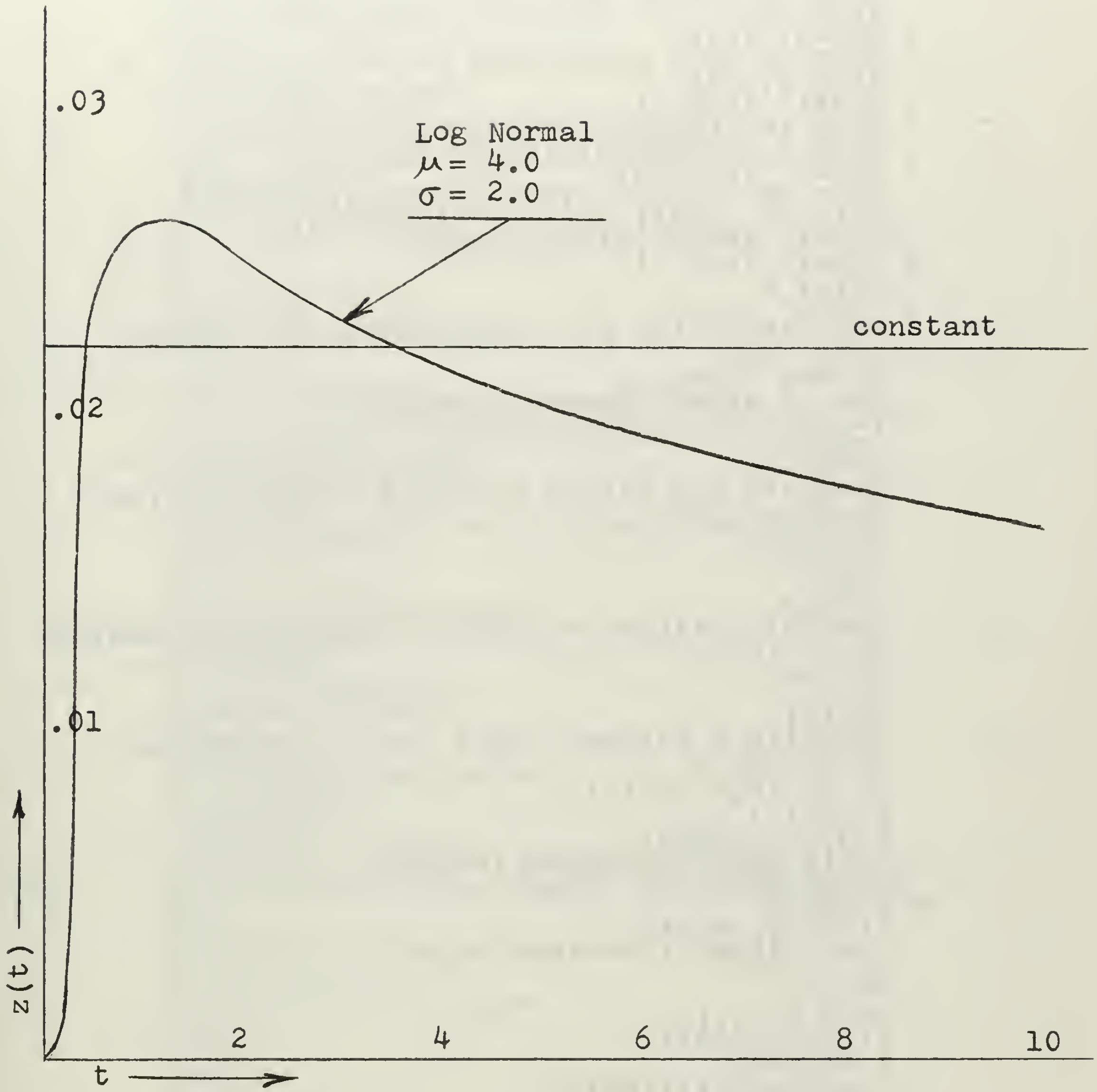

FIGURE 8

Component Failure Rate Functions, z(t), for Case 8.

APPENDIX II

COMPUTER PROGRAM

```
      DIMENSION RS(500),RSL(500),S(4),NF(4),
     *N(4),HLAM(4),BTA(30),TO(4,3)
C     READ IN SAMPLE SIZE, N
      READ 1,N
    1 FORMAT (4I8)
C     READ IN BETA CORRECTION FACTORS
      READ 2, BTA
    2 FORMAT (5F8.3)
C     READ IN PLANNED TEST TIMES
      READ 3, ((TO(I,J),I=1,4),J=1,3)
    3 FORMAT (4F10.0)
C     READ IN FAILURE DISTRIBUTION PARAMETERS
      READ 12, ALFA1,BETA1,ALFA2,BETA2,
     *SIGMA1,XMEAN1,SIGMA2,XMEAN2
   12 FORMAT (8F6.0)
      PRINT 4
    4 FORMAT (1H1)
C     RUN SIMULATION FOR THREE DIFFERENT PLANNED
C     TEST TIMES
      DO 999 IRUN=1,3
C     URN IS RANDOM NUMBER GENERATOR
      Y=URN(0)
      RTOT=0.
      RLTOT=0.
C     GENERATE 500 VALUES OF LOWER CONFIDENCE LIMIT
      DO 50 K=1,500
      HLAMD=0.
      FTOT=0.
      TS=0.
      TSUM=0.
C     SIMULATE FAILURE OF EACH OF THE FOUR COMPONENTS
      DO 30 I=1,4
      S(I)=0.
      NF(I)=0
      M=N(I)
C     GENERATE N FAILURE TIMES FOR EACH COMPONENT
      DO 20 J=1,M
      GO TO (5,6,7,8),I
    5 Y=URN(1)
      BINV1=1./BETA1
      TP=((-ALOG(Y))**BINV1)/ALFA1
      GO TO 10
    6 Y=URN(1)
      BINV2=1./BETA2
      TP=((-ALOG(Y))**BINV2)/ALFA2
      GO TO 10
    7 TS1=0.
      DO 9 IJ=1,12
    9 TS1=TS1+URN(1)
      X1=TS1-6.
      Z1=SIGMA1*X1+XMEAN1
      TP=EXP(Z1)
      GO TO 10
    8 TS2=0.
      DO 11 IJ=1,12
   11 TS2=TS2+URN(1)
      X2=TS2-6.
      Z2=SIGMA2*X2+XMEAN2
      TP=EXP(Z2)
   10 CONTINUE
C     COUNT NUMBER OF FAILURES BEFORE PLANNED
C     TEST TIME EXPIRES
      IF(TP.LT.TO(I,IRUN)) GO TO 15
      T=TO(I,IRUN)
```

```
      GO TO 18
   15 T=TP
      NF(I)=NF(I)+1
   18 S(I)=S(I)+T
   20 CONTINUE
C     CALCULATE LAMDA HAT AND SIMULATED SYSTEM
C     RELIABILITY
      XN=N(I)
      F=NF(I)
      HLAM(I)=(F/S(I))*(2.*XN/(2.*XN+1.))
      FTOT=FTOT+F
      HLAMD=HLAMD+HLAM(I)
      TSUM =TSUM+(HLAM(I)/S(I))
      TS=TS+1./S(I)
   30 CONTINUE
      RS(K)=EXP(-HLAMD)
      RTOT=RTOT+RS(K)
C     CALCULATE UPPER LAMDA HAT AND
C     LOWER CONFIDENCE LIMIT RELIABILITY
      IFTOT=FTOT+1.
      IF(IFTOT-31) 31,32,32
   31 BETA=BTA(IFTOT)
      GO TO 33
   32 BETA=1.
   33 XK=(.842)*BETA
      IF(FTOT) 35,35,40
   35 ULAMD=(XK**2)*TS/4.
      GO TO 41
   40 CHAT=TSUM/HLAMD
      ULAMD=(2.*HLAMD+(XK**2)*CHAT+SQRT(4.*HLAMD*
     *(XK**2)*CHAT+(XK**4)*(CHAT**2)))/2.
   41 RSL(K)=EXP(-ULAMD)
      RLTOT=RLTOT+RSL(K)
   50 CONTINUE
C     CALCULATE MEANS AND STANDARD DEVIATIONS
      RBAR=RTOT/500.
      SUM=0.
      DO 60 K=1,500
      DIFF=(RS(K)-RBAR)**2
      SUM=SUM+DIFF
   60 CONTINUE
      SR=SQRT(SUM/500.)
      RLBAR=RLTOT/500.
      SUM=0.
      DO 61 K=1,500
      DIFF=(RSL(K)-RLBAR)**2
      SUM=SUM+DIFF
   61 CONTINUE
      SRL=SQRT(SUM/500.)
C     ORDER 500 VALUES OF LOWER CONFIDENCE LIMIT
C     ON RELIABILITY AND SELECT 80TH PERCENTILE, A
      II=1
      DO 80 K=1,101
      B=RSL(1)
      DO 70 I=1,500
      IF(B-RSL(I)) 65,70,70
   65 B=RSL(I)
      II=I
   70 CONTINUE
      RSL(II)=0.
      A=B
   80 CONTINUE
C     PRINT OUT VALUES OF MEANS AND STANDARD DEVIATIONS
C     AND 80TH PERCENTILE
      PRINT 90, RLBAR,SRL,RBAR,SR,A
   90 FORMAT(//10X,5F20.5)
  999 CONTINUE
      END
```

APPENDIX III

WEIBULL DISTRIBUTION[6]

The Weibull Probability Distribution is defined by:

$$f(t) = \beta\lambda^{\beta}t^{\beta - 1}e^{-(\lambda t)^{\beta}} \qquad 0 \leq t \leq \infty$$

where λ is the "scale" parameter, and β is the "shape" parameter.

The Cummulative Distribution Function (CDF) is then

$$F(t) = \int_{0}^{t} f(x)dx = 1 - e^{-(\lambda t)^{\beta}}$$

and the failure rate function is

$$z(t) = f(t)/R(t) = \lambda^{\beta}\beta t^{\beta - 1}$$

where $R(t) = 1 - F(t)$ = the reliability function. The Weibull Probability Distribution reduces to the Exponential Probability Distribution when $\beta = 1$. Increasing failure rates are obtained only when $\beta > 1$.

---

[6]Daniel R. Cox, Renewal Theory (London: Methuen and Co. Ltd., 1962) p. 21.

APPENDIX IV

LOG NORMAL DISTRIBUTION[7]

The Log Normal Probability Distribution is given by

$$f(t) = \frac{1}{\sqrt{2\pi}\,\sigma t}\, e^{-\frac{1}{2}\left[\frac{\ln t - \mu}{\sigma}\right]^2} \qquad 0 < t < \infty$$

$$= e^X$$

where X is distributed as Normal (μ, σ^2). For computational purposes, it is beneficial to derive the Cummulative Distribution Function as

$$\begin{aligned} F(t) &= P(\ t \leq T\ ) \\ &= P(\ t \leq e^X\ ) \\ &= P(\ln t \leq X\ ) \\ &= P\left[\frac{\ln t - \mu}{\sigma} \leq Z\right] \\ &= \Phi\left[\frac{\ln t - \mu}{\sigma}\right] \end{aligned}$$

then $$f(t) = F'(t) = \frac{1}{\sigma t}\ \phi\left[\frac{\ln t - \mu}{\sigma}\right]$$

The failure rate function is given by:

$$z(t) = f(t)/R(t) = \frac{\frac{1}{\sigma t}\ \phi\left[\frac{\ln t - \mu}{\sigma}\right]}{1 - \Phi\left[\frac{\ln t - \mu}{\sigma}\right]}$$

---

[7]Paul L. Meyer, Introductory Probability and Statistical Applications (Reading, Mass: Addison-Wesley, 1965) p. 187.

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UNCLASSIFIED

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DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)

Naval Postgraduate School
Monterey, California 93940

2a. REPORT SECURITY CLASSIFICATION

UNCLASSIFIED

2b. GROUP

3. REPORT TITLE

EVALUATION OF THE ACCURACY OF A RELIABILITY MEASUREMENT PROCEDURE USING SIMULATION TECHNIQUES

4. DESCRIPTIVE NOTES (Type of report and, inclusive dates)

Thesis

5. AUTHOR(S) (First name, middle initial, last name)

Kenneth Alan Huffman, Lieutenant, United States Navy

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
December 1968	46	4

8a. CONTRACT OR GRANT NO

b. PROJECT NO

c.

d.

9a. ORIGINATOR'S REPORT NUMBER(S)

9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

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13. ABSTRACT

This study evaluates the accuracy of an established reliability measurement procedure (NAVWEPS OD 29304) by computer simulation. The reliability measurement procedure assumes components fail according to an Exponential Failure Law. This study tests the accuracy of that procedure when components obey a Weibull Failure Law or a Log Normal Failure Law.

DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6811

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A-31408

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Reliability						
Reliability Measurement						
Reliability Confidence Limit						
Weibull Distribution						
Log Normal Distribution						